

Water, Where Are You?

Abstract

At present, the shortage of freshwater resources has seriously restricted the development of various countries in the world, so it is particularly important to develop rational and effective water resources strategies. In order to meet the water demand by 2025, this paper takes China as the research object, analyzes its water resources data, combines with China's economic, ecological and environmental factors and finally establishes freshwater resources strategic planning model.

In order to solve the problems in the water storage and transportation, seawater desalination and water resources protection, we made maximization of social benefits $S(X)$ and economic benefits $J(X)$ as the goal, the water storage, transportation, desalination and water resources protection as the constraint conditions, and set up a multiple objectives programming model. In solving the model, we made the social benefit as the constraint conditions, and then transformed it into a single objective programming model. After the model was solved, the optimal allocation of water resources was achieved. In particular, the application field of the water resources optimal allocation model can be cities, countries and even the whole planet. The model has general applicability. Meanwhile, the model is also applicable to allocation of other resources, such as oil, natural gas, minerals and so on.

In addition, for the model of storage and transportation, we established the optimal model of water resources storage and transportation routes. The amount of water transportation in subdomain K was calculated by using multi-objective programming model mentioned above. Taking the actual different transportation routes into account, according to different transportation costs of different transportation routes, the problem of choosing the best route was converted into the problem of choosing the shortest path. Then the optimal route for water resources storage and transportation was found out by using Dijkstra algorithm. At the same time, for some coastal places where freshwater resources were little, the optimal allocation of water resources is given by comparing the desalination costs with the costs of shortest path of transportation.

For the future water demand till 2025, based on the conditions of population, industry, agriculture, economic and ecological development in different areas, the model of water demand prediction was established. Putting all the data of the future annual water demand into the two models mentioned above, the problem of storage, transportation, protection and utilization of the future water resources was solved and the water resources allocation strategy in future regions were set up.

Finally, taking China as the study area, 10 subdomains were divided according to the characteristics of water distribution. By using the above models, we have gained the amount of future water demand, water resource allocation strategy and cost of strategy implementation. In order to illustrate the application of the model in detail, taking the Haihe River area as an example, the regional demand is relatively large. Putting the number of users, the number of water sources, the number of reservoirs and the relevant data of coastal cities into the three models, the water resources strategy is established.

In 2025, Dan river water can send 1.2 billion cubic meters to Beijing, 1.4 billion cubic meters to Tianjin, 10.8 billion cubic meters to Shijiazhuang, 1.2 billion to Beijing by the Yangzhou branch of the Yangtze River, 1.6 billion by the Yangzhou branch of the Yangtze River, 10.9 billion cubic meters to Qingdao, the Yellow River transport 0.7 billion cubic meters of water to Datong, at the same time, by seawater desalination, Tianjin can gain 200 million cubic meters of water, and a storage capacity of 1 billion cubic meters of reservoir can be built in Shijiazhuang.

Key words: Multi-objective Optimization, Shortest path, Dijkstra algorithm, Water resources planning

Content

<i>I. Introduction</i>	4
<i>II. Description of the Problem</i>	5
2.1 Restatement of the problem	5
2.2 Analysis of the Problem	5
<i>III. Situation of Water Resources in China</i>	6
3.1 China Water Resources Division	6
3.2 Precipitation in China	7
3.2.1 The temporal distribution of precipitation	7
3.2.2 The Special Distribution of Precipitation	7
3.3 Surface water, groundwater and the total amount of water resources	8
<i>IV. Mathematical Models</i>	10
4.1 Assumption	10
4.2 The rational allocation model of water resources	10
4.2.1 The basic principle of the model	10
4.2.2 Quantitative description of objectives	11
4.2.3 quantitative description of constraint condition	13
4.2.4 Model transformation method	15
4.3 Model for water resources reservation and transportation	16
4.4 Water demand forecasting and water supply forecasting model	18
4.4.1 Symbolic description	18
4.4.2 The prediction model of water demand	19
4.4.3 Water Supply Forecast Model	21
<i>V. Model Application</i>	22
5.1 Analysis of Model Application	22
<i>VI Model Conclusion</i>	25
6.1. Strengths	25
6.2 Weakness	25
6.3 Future Work	25
<i>VII. References</i>	26
<i>VIII. Appendix</i>	27

I. Introduction

In most parts of the world, the shortage of freshwater resources seriously limits the development of countries. Rational use of water resources to cope with the future water needs has become a hot issue. In order to understand the problem better, we take China as the study area and establish the optimal water resources strategy. The following background is worth mentioning.

There is plenty of water on the planet which is nearly 1.4 billion km^3 and up to more than 200 million km^3 per person. However, among it, sea water accounts for 97.5 percent, while fresh water makes up merely 2.5 percent, most of which exists in polar ice cap, alpine glacier, and permanent frozen ground as solid form. The amount of fresh water regained from yearly precipitation and evaporation is less than 50 thousands km^3 .

Although fresh water is limited, compared to the early population and social productivity level, it is more than sufficient. For example, the human population was only about 1 billion 200 years ago, and per capita water resource was up to more than 40 thousand m^3 per person. 100 years ago, the human population was about 1.7 billion and per capita water resource was still up to 28 thousand m^3 per person. However, with the continuous expansion of population growth and economic scale, per capita water resource decreases, and meanwhile because various pollutants in water have been increasing, the shortage of water resource and water pollution crisis are intensified.

With over 20 per cent of the world population, China shares merely 6 per cent of the global freshwater resources and its per capita quantity of water resources is less than 30 per cent of that of the world. Plus, the rapid development of city modernization and industrialization in China further worsen water crisis in this country. Such problems as water shortage, water pollution, flood and soil erosion have posted a serious threat to the country's water security and ecological security. Water shortages and serious river pollution in particular have been threatening the country's sustainable economic and social development.

According to the traditional model of development, the water use efficiency and the way of water use, China's water resource and water environment will be difficult to maintain in the near future, let alone sustainable utilization and development.

Therefore, according to the above analysis of background, in order to gradually ease the water crisis, from the perspective of reasonable development, optimal allocation, efficient use and effective protection of water resources, we determine an optimal strategic model of water resources for China.

II. Description of the Problem

2.1 Restatement of the problem

In most parts of the world, freshwater shortages limit various developmental activities. Therefore, a mathematical model needs to be established to realize an effective, feasible and cost-effective water resource strategy in 2013, which can ensure that the expected water demand in 2025 is to be met. The country chosen here is China, for which a best water strategy is to be designed by establishing a mathematical model. With the mathematical model three problems must be solved, namely, water storage and scheduling, desalination and water resources protection. If possible, three factors (i.e. economic, ecological and environmental factors) that determine the strategy need to be explored by the established mathematical model. Moreover, a non-technical document is to be provided for the governmental leaders so as to introduce three aspects, namely, the design method, the feasibility and cost of the proposed strategy, and the reason why it is the best water resources strategy available.

2.2 Analysis of the Problem

There is no denying that the amount of water resources on the planet is abundant. But the amount of fresh water which can be used in human life and production is very little. So in most parts of the world, the shortage of freshwater limits various development activities. Based on the requirements of the subject, we chose China as the research object in several countries given. According to the situation of China's freshwater resource, we combined with China's agricultural, industrial, and life, we design an optimal strategy of water resources, to meet the projected water demand in 2025.

There are three models in the optimal water resources strategy. First model is the model of water resources optimal allocation. On the basis of maximizing the economic benefit, ecological benefit and environmental benefit, we will optimally allocate the water resources, by transferring water from the water-rich regions to water-poor regions. Next, we need to consider how to make up the shortage of fresh water in dry regions. Here we will establish the second model---the model of water resources transportation, to decide which method is better, fresh water transferring or sea water desalinization. Finally, we will build the third model---the model of water demand prediction, which is a forecasting model to forecast the water demand in the year of 2025, and then determine the optimal allocation scheme and transportation scheme of water resources in China's future.

III. Situation of Water Resources in China

With the improvement of productivity and the rapid development of economy, freshwater resources become the earth's most valuable treasure, because it is indispensable for industry, agriculture, daily life and ecological construction. Even though the earth is known as the water planet, freshwater resources take up a very small percentage of its all water resources. Moreover, along with the increase of population, human beings use and waste more water for their own development. If this model of water using continues, freshwater resources might dry up in hundreds of years or even several decades. So we need to design an optimal water resource strategy that ensures a better freshwater allocation to various aspects of the society and higher utilization of freshwater. Here, we'll first analyze the situation of water resources in China.

3.1 China Water Resources Division

In order to objectively reflect the condition of water resources throughout the whole country, the level of water resources development and utilization, the status of water ecological environment, and the interrelationship between the above and the local population, resources, environment and economic and social development, considering the combination of basin and administrative area, and the combination of universality and individuality, the compilation of national unified Water Resources Division [1] should be conducted. Then the evaluation of water resources, water resources allocation, analysis of water resources supplies and demand, water resources protection, water resources planning and other aspects of work should be completed.

The compilation of Water Resources Division should meet the following basic requirements:

- try to maintain integrity of river system.
- can basically reflect the universality in division and individuality between divisions.
- organically combine basin and administrative area, keeping the integrity, modularity and division between them.
- easy to conduct evaluation, planning, exploring and utilization, allocation, protection, management and other work.
- coordinate the relationship between water resources division and other natural resources regionalization.

Based on the requirements mentioned above, China has been divided into 10 large water resources divisions: Songhua River area, Liaohe area, Haihe River area, the Yellow River area, Huaihe area, the Yangtze River area, the Pearl River area, southeastern rivers area, southwestern rivers area, northwestern rivers area. (Distribution of China Water Resources Division see Fig.1.)

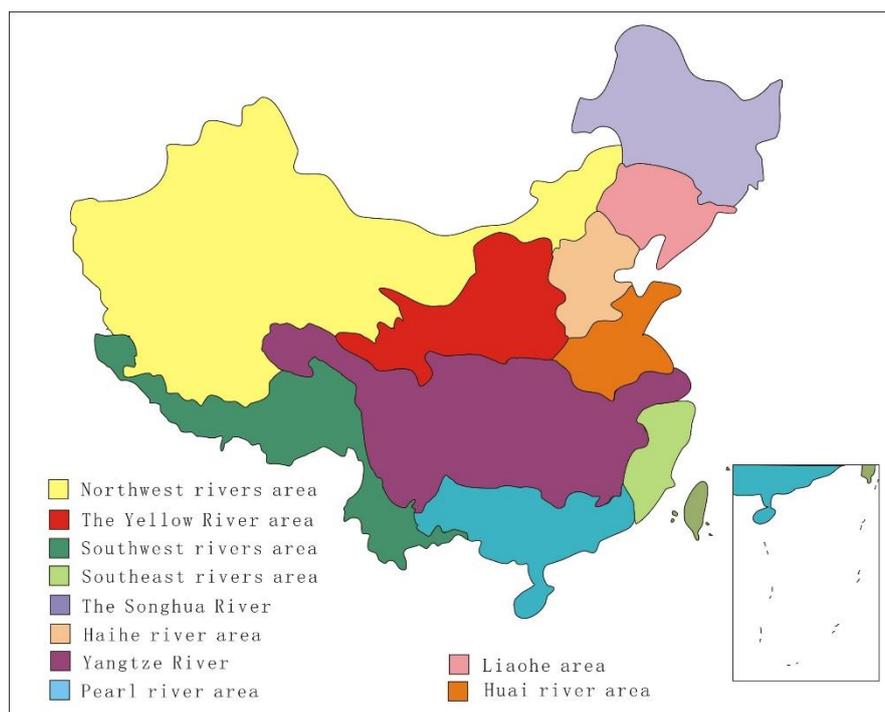


Fig.1.Distribution of China Water Resources Division

3.2 Precipitation in China

3.2.1 The temporal distribution of precipitation

Precipitation in China is mainly controlled by monsoon. It features a noticeable inter-annual variation and a heterogeneity of seasonal distribution. Northwest China has the largest inter-annual variability of precipitation which reads over 0.4. Certain inland dry areas in this region have an even higher variability that reads over 0.6. Normally, the inter-annual variabilities of precipitation of Northeast and Northern China are around 0.3 and that of some places in these regions might reach 0.4. Compared with northern parts of China, Southern China has relatively smaller inter-annual variabilities of precipitation, which generally range from 0.2 to 0.25. The ratio of annual precipitation maximum to minimum in the southern regions of China is about 2 to 3, and that of given areas is up to 4 and even more. As a contrast, the ratio in northern regions of China is 3 to 6 and the highest can reach up to 10.

3.2.2 The Special Distribution of Precipitation

The spatial distribution of precipitation in China is that there is more rainfall in southern China than in northern China and more in eastern China than in Western China. Southern China covers 36 per cent of the area of China, but its precipitation quantity accounts for 67.8 per cent of the country; while Northern China covers 64 per cent of the nation's area, but its precipitation quantity accounts for only 32.2 per cent of the total. By means of collecting data, the distribution of precipitation in 10 divisions of water resources can be seen in Figure 2.

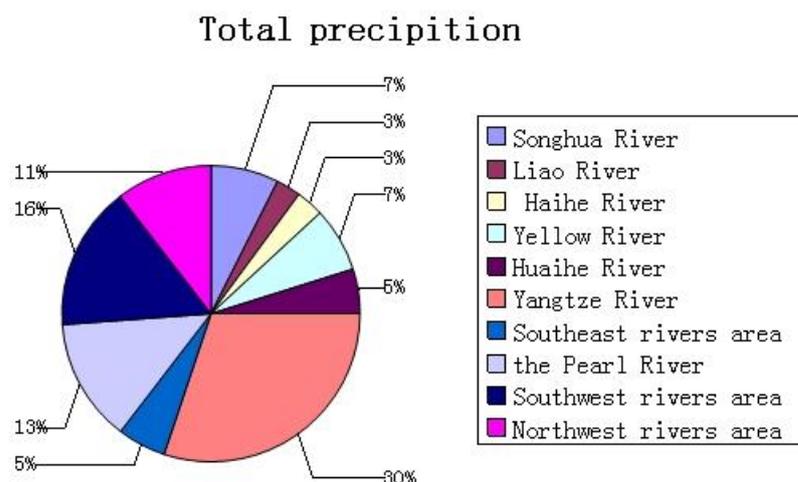


Fig. 2 The distribution of precipitation in 10 divisions of water resources

3.3 Surface water, groundwater and the total amount of water resources

The amount of surface water resources refers to the dynamic content which is within the evaluation range and yearly naturally replenished by local precipitation formation in rivers, lakes, glaciers and other surface water bodies. Corresponding to the spatial distribution of China surface water, the general pattern of China surface water distribution is more surface water in the southern, coastal and mountain areas than in the northern, inland and plain areas. While the amount of groundwater resources is dynamic water by local precipitation and surface water recharge underground aquifers. The characteristics of its spatial distribution are more groundwater in hill areas and the southern areas than in the northern areas and plain areas. The total amount of water resources is within the evaluation range and is the sum of local precipitation formed by surface and underground water. We found out the statistics of surface water, groundwater and total amount of water resources from 2000 to 2011 in the National Bureau of Statistics and drew two diagrams respectively showing total water resources distribution, and the relationship between the amount of surface water resources, the amount of groundwater resources and total amount of water resources. (Total water resources distribution see Fig. 3. The relationship between the amount of surface water resources, the amount of groundwater resources and total amount of water resources see Fig.4)

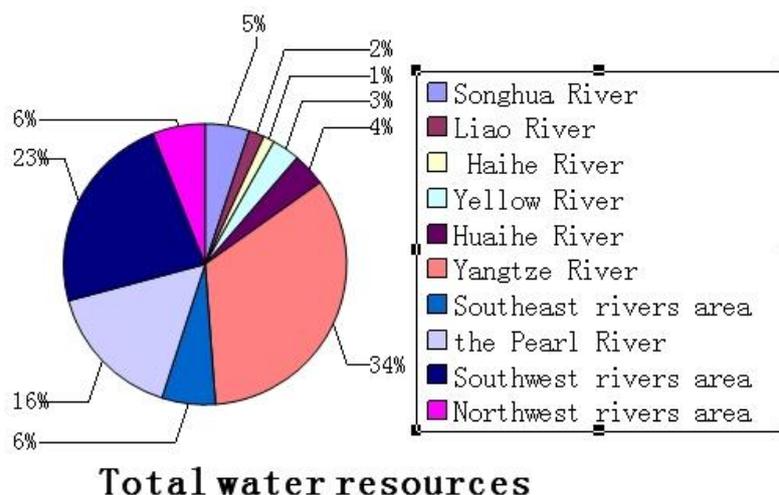


Fig. 3.Total water resources distribution

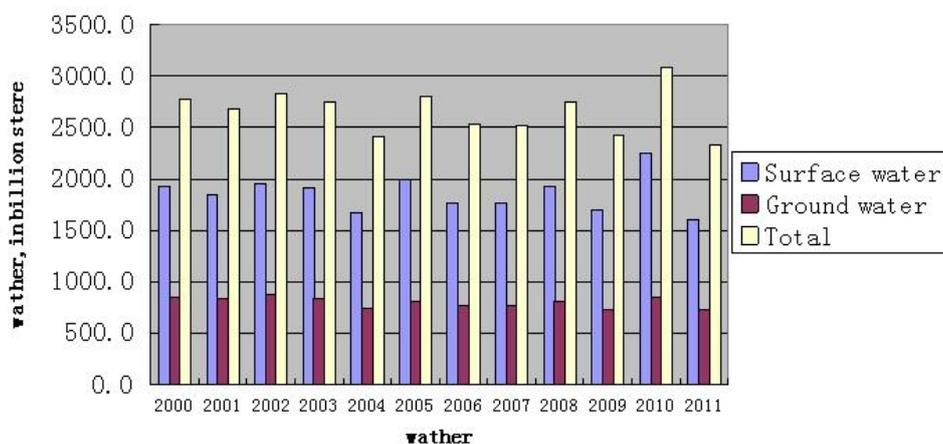


Fig.4 Distribution of surface water, groundwater and the total amount of water resources over the years

3.4 Use of freshwater resources in various fields

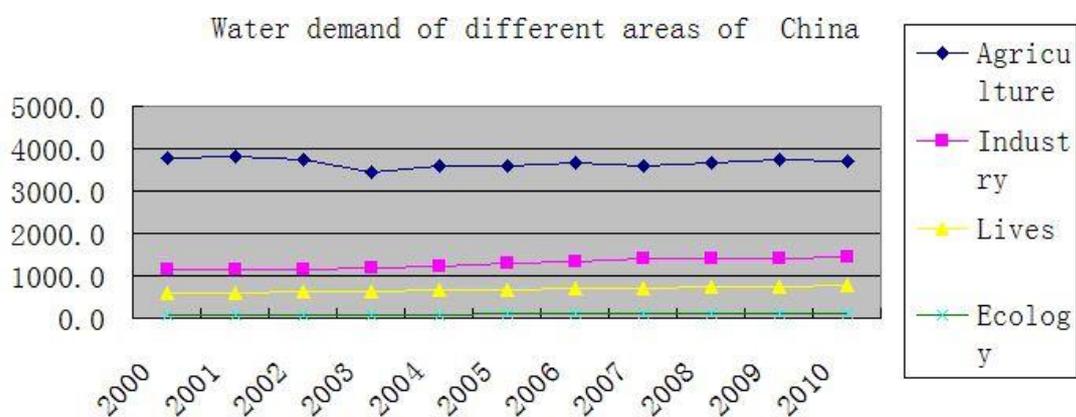


Fig. 5 Use of freshwater resources in such fields as agriculture, industry, daily life and economical construction

As can be seen from Figure 5, agricultural water use takes up the largest share in the overall use of freshwater in China. Industrial water use takes the second place, which is followed by living water use and ecological water use. Here we can see that China is a large agricultural country, but with the development of China's economy, the amount of freshwater used in various areas will change.

IV. Mathematical Models

4.1 Assumption

- From now until 2025, there will be no wars, natural disasters and other influential factors
- Total amount of groundwater resources will not decrease because of permeation, and will not lose in the process of transportation
- The cost of per unit from the same water resource to different regions is the same
- There is no great change in China's economic structure within the next few years
- The desalination cost of per unit volume in different coastal areas is the same
- The coefficients' effects on total amount of water resources or on total amount of water usage is consistent with their effects on the degree of risk.
- Model referencing data is true and effective

4.2 The rational allocation model of water resources

4.2.1 The basic principle of the model

In accordance with the administrative areas, a certain region is divided into K subdomains [5], $k=1,2,3, \dots, K$. Based on the above analysis of characteristics of water resources in China, we choose the country as a large region, and then divide it into 10 smaller areas according to the geographical features. If each study area is viewed as a province, then subdomains represents its every city and town. No matter how study areas are divided, the model of water resources optimization is very applicable. The establishment process of our model is shown in Figure 5.

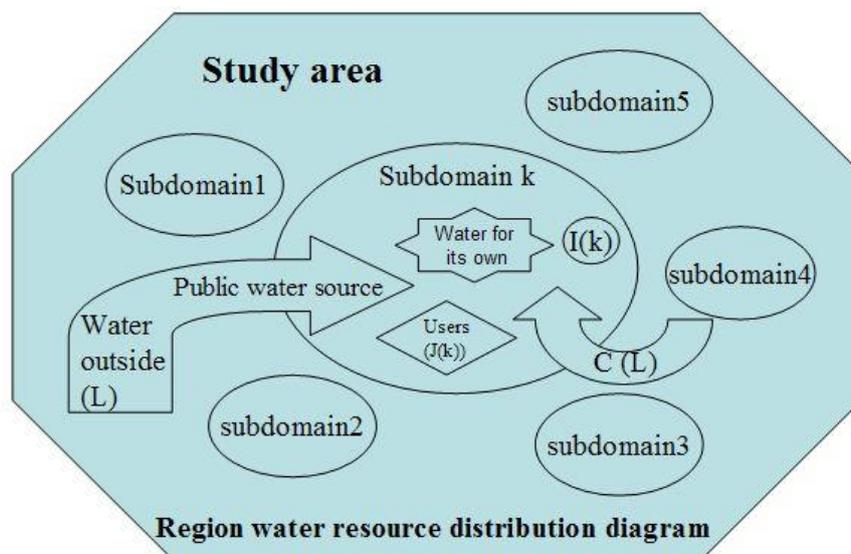


Fig. 6 Allocation of Regional water resources

The denotations for each symbol in Fig. 6 are as follows:

$I(K)$: The total number of internal water sources in subdomain k

$E(k)$: The number of internal reservoirs in subdomain k

$J(k)$: The number of water users in subdomain k

L : the total number of external water sources in all the subdomains of the study area.

M_c^k : The total amount of water resources transported from external water source c to subdomain k ($c=1,2,3, \dots, L$).

Thus, in subdomain i the use of water is the same as that in other water sources, that is, there is a need for distribution of water among all users in this area, and during the distribution process, the prevention and control of water pollution of various subdomains, water transportation costs and the economic benefit brought to each subdomain by water resources should be considered. To ensure that above objectives achieve the optimal effect, we will transform the study of water resource allocation in the region into the study of optimizing multiple objectives [6]. We'll take into account such binding conditions as water demands, water pollution and water conveyance capacity in various areas. If all the above transformed into mathematical problems that would be a multi-objective optimization water allocation of multiple water sources to multiple users.

4.2.2 Quantitative description of objectives

1)Objective 1 Social benefits:

As social benefits involve comprehensive situations in various fields, different points of view may lead to different quantization processes. Here we only consider the

influence of water resources on the society and reckon that levels of water shortage directly affect social development and stability, so we agree on the following symbolic denotations:

- M_j^k : The water demand of user j in domain k(10000 m^3)
- x_{ij}^k :The water quantity supplied by the internal water source i to user j in domain k(10000 m^3)
- x_{cj}^k : The water quantity supplied by the external water source c to user j in domain k (10000 m^3)
- x_{ej}^k : The water quantity supplied by the reservoir e to user j in domain k(10000 m^3)
- $S(X)$: The economic benefits brought to the whole area by optimization of water resources allocation

Our allocation strategy aims to minimize the shortage of water in this area for each user, thereby establishing the goal formula:

$$\max S(X) = -\min\left\{\sum_{k=1}^K \sum_{j=1}^{J(k)} [M_j^k - (\sum_{i=1}^{I(k)} x_{ij}^k + \sum_{c=1}^L x_{cj}^k + \sum_{e=1}^{E(k)} x_{ej}^k)]\right\}$$

2) Objective 2 economic benefit:

Economic aspects are related to costs of water resources transportation. Costs of different scheduling paths will be different. At the same time, scheduling and allocation of water resources will bring economic benefits to the user. Thus the result is calculated as the biggest by subtracting the cost of water resources transportation from the economic benefit brought by the water resources. However, with the matter of freshwater resources getting worse gradually, the value of freshwater is much higher than before. What's more, exploitation and usage of surface water and groundwater is excessive. All of these force us to find new means to get more freshwater resources. Here we present the alkalization and desalination method to coastal seawater. For some specific coastal cities, we are able to desalinate seawater, but this process will produce additional cost compared to other users. In order to unify, we consider adding the variable 0,1 to the coastal zone users. So we can sort out the second objective functions. Here we add the following denotations of symbols:

- b_{ij}^k, b_{cj}^k : Economic value of per unit water supply from the internal water source i and external water source c to subdomain j (RMB/ m^3)
- δ : Decision variable 0,1 of coastal users

- c_{wj}^k : The total cost of water resources consumption
- c_{dj}^k : Customer j's cost of per unit seawater desalination in subdomain k
- c_{ej}^k : Customer j's cost of per unit water resources supply in reservoir e (RMB/ m^3)
- c_{ij}^k 、 c_{cj}^k : Costs of customer j unit supply of water from internal water source I and external water source j to subdomain j (RMB/ m^3)
- a_{ij}^k 、 a_{cj}^k : Customer j's adjustment factors of supplying benefit from internal water source I and external water source c
- β_j^k : Level coefficient of customer j in subdomain k
- ω_k : Weight coefficient in subdomain k
- $J(X)$: Total economic benefit in the study area

From The above analysis, we can quantitatively express the economic benefit formula:

$$c_{wj}^k = c_{ij}^k + c_{ej}^k + \delta c_{dj}^k$$

$$\max J(X) = \max \left\{ \sum_{k=1}^K \sum_{j=1}^{J(k)} \left[\sum_{i=1}^{I(k)} (b_{ij}^k - c_{wj}^k) x_{ij}^k a_{ij}^k \beta_j^k \omega_k + \sum_{c=1}^L (b_{cj}^k - c_{wj}^k) x_{cj}^k a_{cj}^k \beta_j^k \omega_k \right] \right\}$$

$$\delta = \begin{cases} 1 & , \text{ customer belongs to the coastal areas?} \\ 0 & , \text{ customer does not belongs to the coastal areas} \end{cases}$$

4.2.3 quantitative description of constraint condition

1) Water supply capacity constraints

Economic development is different in different regions. exploitation and utilization degree of the water resource is different. both from the perspective of technology and cost of water supply Water supply capacity constraints has a maximum limit, so we add the following notational conventions (other symbolic significance and the Convention)

- N_c : External water source C's water supply cap
- N_i^k : Water source I's water supply cap in subordinate k
- W_i^k : Water supply cap of an internal water I in subordinate K
- M_e^k : Water supply cap of reservoir e in subordinate K

From the perspective of the three water supply caps, considering the supply constraints, which is internal water source limits in each subdomain, restrictions on the quantity of water of each reservoir in each subdomain, and constraints on total amount

of water from external water sources, we get the following constraint formula

$$\left\{ \begin{array}{l} \sum_{j=1}^{J(k)} x_{ij}^k \leq W_i^k \\ \sum_{j=1}^{J(k)} x_{cj}^k \leq M_c^k \\ \sum_{j=1}^{E(k)} x_{ej}^k \leq M_e^k \\ \sum_{k=1}^K M_c^k \leq N_c \end{array} \right.$$

2) constraints on filling and emptying capacity

Based on the constraints, we found that different sources of water supply may not be able to transport its water to the area, which has a capacity constraint.

- Q_c : The maximum carrying capacity for External water c.
- Q_i^k : The maximum carrying capacity for internal water i in subdomain.
- Q_e^k : The maximum carrying capacity for reservoir e

$$\left\{ \begin{array}{l} x_{cj}^k \leq Q_c \\ x_{ij}^k \leq Q_i^k \\ x_{ej}^k \leq Q_e^k \end{array} \right.$$

3) Constraint of User demand and supply of water change

According to the above analysis, there is a big difference on the different user groups in their water requirements, such as residents' living water, industrial water, agricultural water. So here we need to add a constraint on user's water requirement

- M_{jL}^k, M_{jH}^k :Upper and lower bounds for user j in subdomain k separately.

$$M_{jL}^k \leq \sum_{i=1}^{I(k)} x_{ij}^k + \sum_{c=1}^L x_{cj}^k + \sum_{e=1}^{E(k)} x_{ej}^k \leq M_{jH}^k$$

4) constraint on the sewage drainage quantity of drainage system

Related to environmental issues, the main source of water pollution is the waste of water resources. In different users' water scheming in the different areas, users must take into account the issue of environmental protection. Here we use the content of [5], which is a major contaminant in wastewater discharge. To represent it, we can use chemical oxygen demand COD, biochemical oxygen demand BOD (mg/L), so we were on the water pollution limit constraints. Constraint conditions are as follows.

- Z_{kj}^s : Concentration of pollutant s discharged by user j in subdomain k.

- z_0^s : Concentration of pollutant s discharged by user j in subdomain k when meet discharging criterions.
- N_0 : Total amount of pollutants discharged allowed
- d_j^k : Content of important pollutants such as COD, BOD in unit waste water discharged by user j in subdomain k .
- p_j^k : Emission coefficient of sewage for user j in subdomain k .

$$\begin{cases} z_{kj}^s \leq z_0^s \\ [\sum_{k=1}^K \sum_{j=1}^{J(k)} 0.01 d_j^k p_j^k (\sum_{i=1}^{I(k)} x_{ij}^k + \sum_{c=1}^L x_{cj}^k)] \leq N_0 \end{cases}$$

5) Nonnegative constraints

The demand is not less than zero. of course the external water sources we introduced from outside have different meanings, whether it is positive or negative, big or small. greater than zero indicates that the user needs to be transferred of water resources from outside; less than zero indicates the area that needs to transfer water resources outside; with the non-negative constraints of reservoir water scheming, allocation of internal water resources is nonnegative constrained:

$$\begin{aligned} x_{ij}^k &\geq 0 \\ x_{ej}^k &\geq 0 \end{aligned}$$

4.2.4 Model transformation method

Water resources strategy for us on the bi-objective optimization model has been established, but the solution of the multi-objective optimization model, method is commonly used many target weights increase the proportion, which is transformed into a single objective problem. In this model we established we use is the social benefit target which is processed, the shortage of the research area all users with the given a limit value, the objective function into constraints

- Q_s : Limit user shortage of regional total value

$$\sum_{k=1}^K \sum_{j=1}^{J(k)} [M_j^k - (\sum_{i=1}^{I(k)} x_{ij}^k + \sum_{c=1}^L x_{cj}^k + \sum_{e=1}^{E(k)} x_{ej}^k)] \leq Q_s$$

So our bi-objective optimization model can be transformed into a single objective optimization problem.

$$\max J(X) = \max \left\{ \sum_{k=1}^K \sum_{j=1}^{J(k)} \left[\sum_{i=1}^{I(k)} (b_{ij}^k - c_{wj}^k) x_{ij}^k a_{ij}^k \beta_j^k \omega_k + \sum_{c=1}^L (b_{cj}^k - c_{wj}^k) x_{cj}^k a_{cj}^k \beta_j^k \omega_k \right] \right.$$

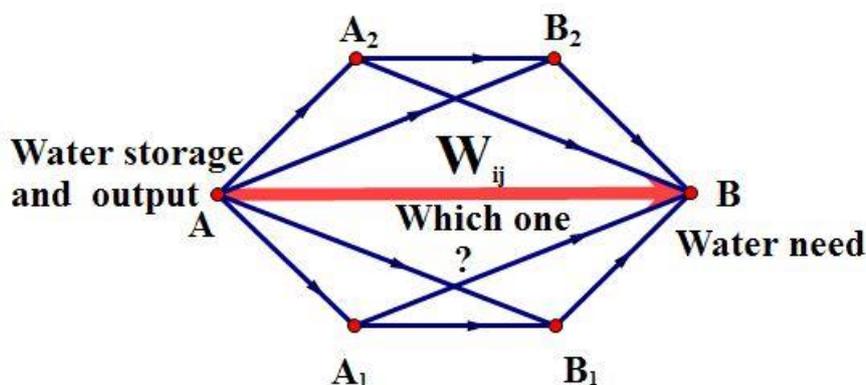
$$\left. \begin{array}{l} \sum_{k=1}^K \sum_{j=1}^{J(k)} [M_j^k - (\sum_{i=1}^{I(k)} x_{ij}^k + \sum_{c=1}^L x_{cj}^k)] \leq Q_s \\ \sum_{j=1}^{J(k)} x_{ij}^k \leq W_i^k \\ \sum_{j=1}^{J(k)} x_{cj}^k \leq M_c^k \\ \sum_{k=1}^K M_c^k \leq N_c \\ M_{jL}^k \leq \sum_{i=1}^{I(k)} x_{ij}^k + \sum_{c=1}^L x_{cj}^k + \sum_{e=1}^{E(k)} x_{ej}^k \leq M_{jH}^k \\ x_{cj}^k \leq Q_c \\ x_{ij}^k \leq Q_i \\ z_{kj}^s \leq z_0^s \\ [\sum_{k=1}^K \sum_{j=1}^{J(k)} 0.01 d_j^k p_j^k (\sum_{i=1}^{I(k)} x_{ij}^k + \sum_{c=1}^L x_{cj}^k)] \leq N_0 \\ x_{ij}^k \geq 0 \end{array} \right\} \text{ s.t.}$$

To the above analysis, the single objective optimization model we can use LINGO to solve, so that we on the distribution situation of water diversion of users in every area.

4.3 Model for water resources reservation and transportation

In the reasonable water resources allocation model, we consider the social benefit, economic benefit and ecological environment benefit, we get the water resource distribution in water area. But how can we solve water area for transportation? Because of China's vast territory, rivers of various sizes of complex distribution of water resources in general, relatively large, but the regional distribution is extremely uneven, so the water resource cross-domain transport is a big project, at present has started the South-to-North Water Diversion Project, which involves the transport path selection problem, the transportation of different, and each region the reservoir distribution, its cost is not the same. Water cost allocation model of water resources allocation scheme we give water involves depends on the choice of path, in order to explain our solution, we will be two water diversion areas China is simplified to the vertex, which is now a need from A to transport to the total content of B, but there are different routes of AB between the two places, perhaps halfway through the different points, each line of the underground pipe laying cost, because of geographical differences have different differences between different regions, the transportation route into a weighted graph,

the simple abstract diagram as follows:



the Best route choosing of water storage and movement

Fig. 7 Weighted graph of transportation between the two regional routes

Fig. 7 each edge weight is the main basis for the transport of the distance between two points, and the line unit distance transportation cost and the cost of pipeline laying, geographical features of practical problems in China, water distribution and peak distribution is relatively complex, if the water resource distribution between two points is the shortest distance, but there may be the mountain barrier between them, with respect to the pipeline laying by other routes, underground pipe laying mode will consume cost more, so the following distribution we give each edge weights of the graph:

W_{ij} : The weight of each edge in the graph weights

d_{ij} 、 q_{ij} : Respectively I, J units between the distance between two points and the distance water transportation costs.

$$W_{ij} = d_{ij}q_{ij}$$

So our problem is transformed to weighted minimum paths in graph [7], now that we have m each node (equivalent to A in Figure 2, the B points), n arcs network $N(V, E, W)$, so that we can establish the following integer programming model:

$$\begin{aligned} & \min \sum_{(i,j) \in E} w_{ij} s_{ij} \\ & \text{s.t.} \begin{cases} \sum_{(i,j) \in E} s_{ij} - \sum_{(k,i) \in E} s_{ki} = \begin{cases} 1 & (i=1) \\ 0 & (i=2, \dots, m-1) \\ -1 & (i=m) \end{cases} \\ s_{ij} = 0 \text{ or } 1, (i, j) \in E \end{cases} \end{aligned}$$

We use the Dijkstra shortest path algorithm to solve the model, algorithm steps are as follows:

Step1 : Beginning form fixed label $p(v_1)=0$, the rest of the temporary label

$$T(v_j) = +\infty, j \neq 1.$$

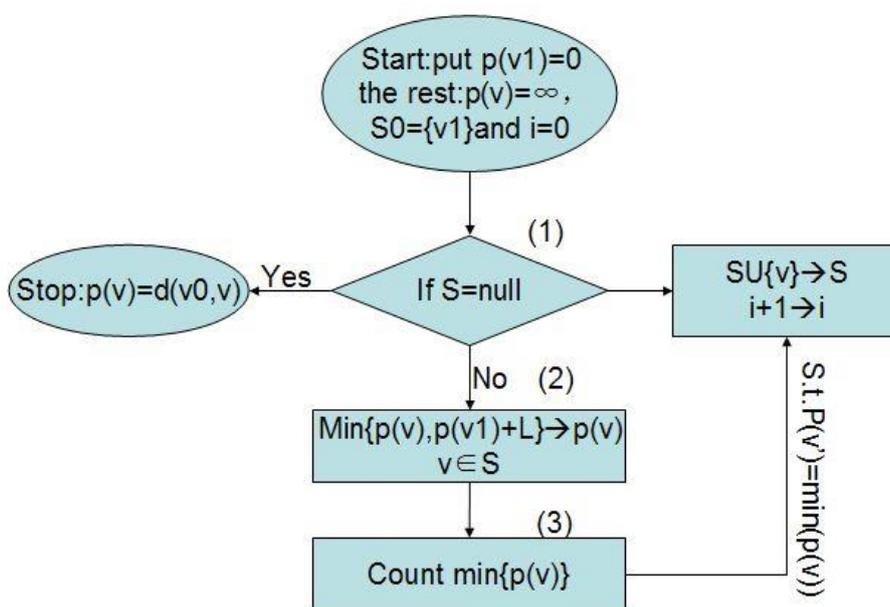
Step2: A node v_i is just got P grade point, consider the point v_j , and $(v_i, v_j) \in E$ as the

T label. The T label for the following changes: $T(v_j) = \min\{T(v_j), p(v_i) + l_{ij}\}$.

Step3: Compared with the T label all nodes, the smallest change as the P label, i.e.

$$T(v_k) = \min[T(v_i)].$$

When there is a minimum of two or more persons, can be changed to P label. If all the nodes are P label, stop, or instead of, return to step (2).The algorithm flow chart is shown in Figure 7.



Dijkstra algorithm operation flow chart

Fig. 8 Dijkstra algorithm

4.4 Water demand forecasting and water supply forecasting model

4.4.1 Symbolic description

W_i^t	Future water demand prediction
GLW_i^t	Life gross water demand prediction
GAW_i^t	Agricultural irrigation water demand prediction of hair
GIW_i^t	Industrial gross water demand prediction

GCW_i^t	The construction industry gross water demand prediction
GTW_i^t	The tertiary industry gross water demand prediction

4.4.2 The prediction model of water demand

According to the requirements of the subject, our goal is to be able to design a long-term effective, can carry out the best strategy of water resources, in order to meet the water demand is expected to 2025. So our idea is based on the Chinese freshwater resources present condition, and to consider the economic development, population growth and environmental factors, we first establish a time and water demand.

Water quantity of total demand:

$$W_i^t = GLW_i^t + GAW_i^t + GIW_i^t + GCW_i^t + GTW_i^t$$

1) Prediction of water demand quantity

As the population increase year by year, the development of social economy, the rate of city with an annual increase of 1.35% growth rate, so we establish the urban residents and rural residents living water model are as follows

Life net water requirement: $LW_i^t = Po_i^t \times LQ_i^t \times 365 / 1000$

Life gross water demand: $GLW_i^t = LW_i^t / \xi_i^t$

In the formula, i as the user classification number, $i=1$ urban, $i=2$ rural; t for the planning level years number; LW_i^t for the i user the t planning level years annual water demand (million *cum*); Po_i^t for the i user the t planning level years of water for the population, LQ_i^t the average daily water consumption of the t year (L / person * days), ξ_i^t coefficient the i user the t planning water use, determined by the living water supply system.

Domestic water demand years relatively uniform, can be calculated the average monthly water to determine the annual domestic water process. Team in water use amplitude larger areas, through typical investigation and analysis of water consumption, determine the required water distribution coefficient, and years of domestic water process of life water requirement determination.

2) Agricultural water demand forecasting

Farmland irrigation water requirement: $AW_i^t = \sum_{i=1}^n (A_i^t \times AQ_i^t)$

Irrigation of farmland gross water demand: $GAW_i^t = AW_i^t / \omega_i^t$

i For the farmland classification number of type.

A_i^t as the number of farmland the planning of farmland.

AQ_i^t : farmland irrigation of farmland for No. the planning of water demand.

ω_i^t : denotes the water utilization rate.

3) Prediction of industrial water demand

$$IQ_i^{t_2} = (1 - \delta)^{t_2 - t_1} \times \frac{1 - \eta_i^{t_2}}{1 - \eta_i^{t_1}} \times IQ_i^{t_1}$$

In the formula, i for the industrial sector classification number; $IQ_i^{t_2}$ and $IQ_i^{t_1}$ respectively t_2 and t_1 the planning level years the i industry water quota (select million yuan output value of water consumption), δ for the comprehensive influence factor, including the progress of science and technology, the product structure and other factors; $\eta_i^{t_2}$ and $\eta_i^{t_1}$ respectively t_2 and t_1 the planning level years bottom i Industrial water recycle rate.

Net industrial water demand:
$$IW_i^t = \sum_{i=1}^n (X_i^t \times IQ_i^t)$$

Industrial gross water demand:
$$GIW_i^t = IW_i^t / \eta_i^t$$

IW_i^t For the planning level years net industrial water demand gross type, X_i^t as the i industry department in the t planning level years industrial development indicators (such as the total output value or value added, such as power generation, for the industrial sector) IQ_i^t the t planning level years of water for i industrial use, GIW_i^t as the level of t planning industrial gross requirement, coefficient of the industrial sector in the t planning of water use, η_i^t as determined by the i industrial t water supply system.

4) The construction industry water demand prediction

The construction industry net water requirement:
$$CW_i^t = B_i^t \times CQ_i^t$$

The construction industry gross water demand:
$$GCW_i^t = CW_i^t / \zeta_i^t$$

B_i^t Type of unit building area, CQ_i^t water consumption for unit construction area, ζ_i^t

construction industry water utilization coefficient.

5) The tertiary industry water demand prediction

For the prediction of the tertiary industry water demand we use million increase in the value of water quota method.

The tertiary industry net water requirement: $TW_i^t = p^t \times TQ_i^t$

The tertiary industry gross water demand: $GTW_i^t = TW_i^t / \delta_i^t$

p^t Value of the tertiary industry in the formula (million), TQ_i^t added value of the tertiary industry million yuan, δ_i^t the tertiary industry water use coefficient.

6) Ecological Water Demand Prediction

Calculation method of water demand for ecological environment we're here to vegetation as the main body, the ecological and environmental water requirement of a vegetation type, calculation methods can be used in a water table multiplied by the area under the diving evaporation and vegetation coefficient, calculation formula is:

$$WST = \sum_{i=1}^n A_i \cdot Wg_i \cdot K$$

A_i : Square

i : Vegetational form

K : Vegetation coefficient

Wg_i : Eevaporation discharge of phreatic water

4.4.3 Water Supply Forecast Model

Water supply refers to the water requirements, different years, different guarantee rate, have been built or planned water supply project may provide content.

$$Q^t = Q_1^t + Q_2^t + Q_3^t + Q_4^t$$

Q_1^t : The surface water supply, Q_2^t : underground water supply, Q_3^t : for the water supply of reservoir, Q_4^t : water supply for the accumulation of rain.

V. Model Application

5.1 Analysis of Model Application

Based on the establishment of these three models, we take China as the study area. According to the previous data about China water resources, combined with China's national conditions and China's future economic development, adopting the strategic thought of our three models, our models are used to analyze the future water demand of each field, and to conduct optimal allocation, reservation and transportation.

Based on the ideas of the first model, we divided China into ten subdomains. Because China is vast in territory, in order to illustrate our water allocation strategy in detail, we used our model again in each subdomain and found out each callable water distribution point in subdomains, as shown in Fig. 9



Fig. 9 the distribution of the callable water resources points in subdomains

According to the China water conservancy statistics yearbooks which last 12 years, we collected the amount of water demand at each allocation points, the amount of water transferred from the internal and external allocation points, the reservoir capacity of internal and external reservoirs, and pollutants concentration of per unit volume of water in the area (Appendix 2), to solve the data into our model for multiple objective programming, combined with our third model, we can get the distribution of the future amount of transportation and allocation, as shown in table 1. (only part of the users' areas are listed)

Water out	Planning Level Years	Beijing	Tianjin	Shijiazhuang
Danjiangkou	2015	10.156	13.386	103.122
	2020	15.081	15.141	105.243
	2025	12.122	14.578	108.811
Yangzhou		Beijing	Tianjin	Shijiazhuang
	2015	18.231	15.361	107.784
	2020	14.511	17.561	106.577
	2025	12.079	16.256	109.121
Yellow River		DaTong		
	2015	8.901	No Data	No Data
	2020	5.982		
	2025	7.936		

Tab.1. distribution of the future amount of transportation and allocation (Unit: 100million cu.m)

The region shown between water transfer we have allocated, but the specific route has not yet given, we use the model of two, to choose the best path to save the transportation cost on the edge weights, we selected the unit cost of comprehensive water route and its route length, and its weight ratio will be an integer value between 1 to 10

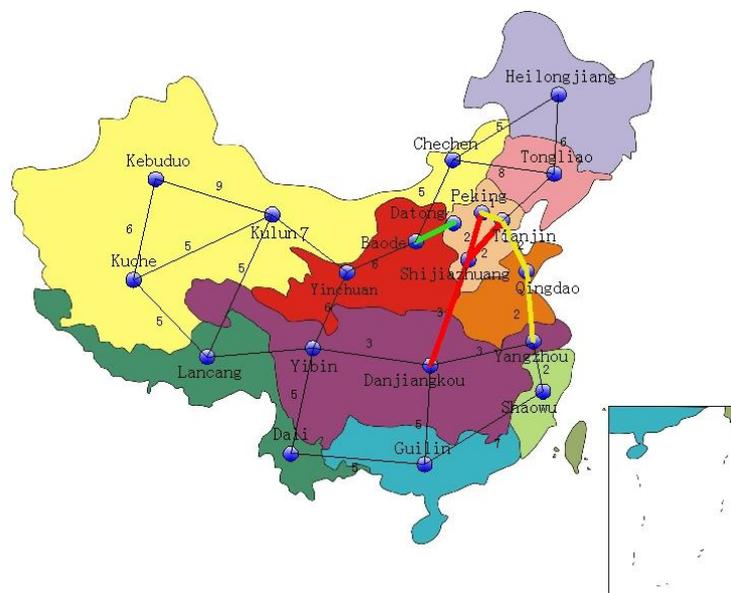


Fig.10. Optiomization map for selection of routes in water allocation

Thick lines above part is Baode and Datong, Danjiangkou and Peking, the optimal path between Yangzhou and Peking, two water diversion point of minimum cost, the water diversion route selected for each path, the distance between the (Appendix 2), the completion of our calling route selection scheme, using model two, calculate the weight

of each the total value of the path, the 4 conveyance path above we choose, the distance between as shown in the following table.

Water Source	Beijing (km)	Tianjing(km)	Shijiazhuang (km)
Danjiangkou	1113	1114	844
Yangzhou	992	956	
Yellow	449.8		

Tab.2. The optimal distance of water sources transportation
(the others shall be shown in appendix 2)

River section within the same assumptions were identified according to our estimate of the different path segment transportation fees (Appendix 2) and then calculate the total path cost is approximately: 49 billion yuan.

In the model three future water demand forecast model, search for Chinese regional agriculture, through the statistics of industrial, life, water for Ecology (Appendix 3), and the development trend in the future, we can expect in every field in China future water demand is the water used as shown in table 2:

Level year	Water supplied	Agriculture	Industry	Lives	Ecology	Totals
2015	19214.2012	3826.252 2	1683.292	867.5428	103.5872	6474.2
2020	20857.126	4019.652 3	1985.273 3	989.1291	161.3473	7015.1
2025	23654.262	3928.157 4	2214.186 1	1022.497 9	191.2586	7356.1

Tab 3. Prediction of water demands in different fields in the future China
(Unit: 100million cu.m)

So we provide model, combined with the use of all, in-depth analysis of China's future water conditions, the above results can be found in our budget, Beijing, Tianjin, Shijiazhuang, Haihe River area city of the future is the need for external transport over the water, for each city, each city such as Beijing, the future introduction of foreign sources are different, according to our optimization model, combined with the prediction model to predict future demand situation, we have to mobilize the situation of regional water each divided region, such as the Haihe River area from 2015 to 2020, the future, and then to 2025, on the basis of our optimization model respectively from the Dan river water, Yangzhou, the Yellow River and other regions to mobilize different content, in order to meet the water demand in the future 2025.

At the same time optimization model we considered in the coastal city of seawater desalination, previously adequate water resources makes the desalination of sea water is relatively high cost, but the current Chinese economic development, water resources value increase, sea water desalination attention, here we take into account the economic cost of desalination of sea water, the conclusion of the Haihe River area optimization,

finally combined with the optimization of path selection, relates to the cost of water resources transportation costs, we expect our water strategy implementation cost: 490000000000 yuan. And is expected next 2025 out of China in all areas of water demand (Table 2), and water diversion scheme (Table 1), the rational use of water resources, protection of water resources strategy to deal with the water demand in various fields.

VI Model Conclusion

6.1 Strengths

- Water resources strategy model we designed can be converted into an oil resources strategy model, scheduling and allocating oil resources by changing parameters of the model.
- Water resources strategy model we designed has brought social benefit, economic benefit, and other factors into consideration so that it is more reasonable and economical. In the solving process, by adding the restriction factor, we skillfully transformed the multi-objective problem into single objective problem, to simplify calculation.
- Water resources strategy model is universal. it is not only suitable for China, but also for any other regions or countries.
- The future water demand model we designed has brought factors in the fields of agriculture, industry, construction industry, the tertiary industry and ecology into consideration. Therefore it can accurately predict the future water demand.
- When we designed the model to supply water resources shortage region with water, method for the shortest path was used. It can reduce the cost of transporting water and make our model more economical.

6.2 Weakness

- Water resources strategy was established under the conditions: the annual amount of water usage of residents in the same area and the annual population growth rate were unchanged.
- Water resources strategy did not bring changes in future economic structure, natural disasters and other influential factors into consideration.

6.3 Future Work

Water resources strategy we build embraces three models: the model of water resources optimal allocation, the model of water resources transportation and the model of water demand prediction. Through the three model, we can apply the three models to optimizing the allocation of other resources. For the applying field of the Water resources strategy, we can apply the strategy in any country, or in the water resource scheduling problem between the various countries in the world. In addition, for the distribution object, we can change the relevant coefficients and transform it into a

distribution strategy of oil or other resources, such as natural gas, scheduling and allocation of mineral resources.

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VIII. Appendix

List 1: Water resources data of ten subdomains in China

Unit : 100billion cu.m	Water supplied	Water need	Water output	Limited of reservoir	Weather near the sea	COD(10thousand t/a)	NH3- N(10thousand t/a)
Heilongjiang	75	45	30	16	0	15.3	1.3
Tongliao	27.5	27	0.5	12	0	12.5	0.9
Chechen	34	33	1	15	0	12.1	0.6
Peking	40.973	42.194	-1.221	14	1	15.7	1.05
Tianjin	21.927	31.874	-9.947	12	1	19.88	0.84
Shijiazhuang	102	187	-85	9	0	55.4	4.53
Datong	13.4	17.6	-4.2	9	0	3.87	0.61
Qingdao	27	32	-5	8	1	21.9	2.77
Baode	44	32	12	18	0	1.6	0.8
Yinchuan	7.4	7	0.4	16	0	2.5	0.7
Kulun	12	11	1	17	0	0.7	0.7
Kebuduo	25	22	3	13	0	0.9	1.2
Kuche	36	34	2	20	0	1.2	1.3
Lancang	31	25	6	14	0	0.9	0.7
Yibin	258	243	15	65	0	5.4	2.3
Danjiangkou	195	65	130	15	0	4.3	0.7
Yangzhou	287	32	255	55	1	6.7	1.3
Dali	35	33	2	35	0	3.2	1.5
Guilin	36	31	5	65	0	0.4	0.01
Shaowu	55	34	21	45	1	2.5	0.3

List 2: POPULATION

Year	Total population	Urban		Rural	
		population	ratio(%)	population	ratio(%)
1980	98705	19140	19.39	79565	80.61
1981	100072	20171	20.15648733	79901	79.84351267
1982	101654	21480	21.13050151	80174	78.86949849
1983	103008	22274	21.62356322	80734	78.37643678
1984	104357	24017	23.01426833	80340	76.98573167
1985	105851	25094	23.71	80757	76.29
1986	107507	26366	24.52491466	81141	75.47508534
1987	109300	27674	25.31930467	81626	74.68069533
1988	111026	28661	25.81467404	82365	74.18532596
1989	112704	29540	26.21024986	83164	73.78975014
1990	114333	30195	26.41	84138	73.59
1991	115823	31203	26.94	84620	73.06
1992	117171	32175	27.46	84996	72.54
1993	118517	33173	27.99	85344	72.01
1994	119850	34169	28.51	85681	71.49
1995	121121	35174	29.04	85947	70.96
1996	122389	37304	30.48	85085	69.52
1997	123626	39449	31.91	84177	68.09
1998	124761	41608	33.35	83153	66.65
1999	125786	43748	34.78	82038	65.22
2000	126743	45906	36.22	80837	63.78
2001	127627	48064	37.66	79563	62.34
2002	128453	50212	39.08978381	78241	60.91021619
2003	129227	52376	40.53022975	76851	59.46977025
2004	129988	54283	41.76000862	75705	58.23999138
2005	130756	56212	42.98999663	74544	57.01000337
2006	131448	58288	44.34286667	73160	55.65713333
2007	132129	60633	45.889	71496	54.111
2008	132802	62403	46.98913333	70399	53.01086667
2009	133450	64512	48.342	68938	51.658
2010	134091	66978	49.95	67113	50.05

List 3:

Year	Urban domestic water consumption	Rural life water	Farmland real irrigation area	Ten thousand yuan industrial added value
1998	222	87	488	
1999	227	89	484	
2000	219	89	479	288
2001	218	92	479	268
2002	219	94	465	241
2003	212	68	430	222
2004	212	68	450	196
2005	211	68	448	
2006	212	69	449	
2007	211	71	434	131
2008	212	72	435	108
2009	212	73	431	103
2010	193	83	421	90

List 4: 2005 The total

Year	Total water resources	Available water	Water amount $10^8 \cdot m^3$
2000	27701	21061.0703	5497.6
2001	26868	20427.7404	5567.4
2002	28255	21482.2765	5497.3
2003	27460	20877.838	5320.4
2004	24130	18346.039	5547.8
2005	28053.1	21328.77193	5633.0
2006	25330.1	19258.47503	5795.0
2007	25255.2	19201.52856	5818.7
2008	27434.3	20858.29829	5910.0
2009	24180.2	18384.20606	5965.2
2010	30906.4	23498.13592	6022.0

List 5: Tian jin

Year	total water resources	Available water
	$10^8 \cdot m^3$	$10^8 \cdot m^3$
2001	38.9	14.208
2002	34.6	11.914
2003	35.8	13.616
2004	34.6	15.836
2005	34.5	17.168
2006	34.3	16.354
2007	34.8	17.612
2008	35.1	25.308
2009	35.5	26.27
2010	35.2	17.094

List 6: Tian jin

Year Region	Water Supply & Use
2002	199610
2003	205118
2004	219623
2005	225193
2006	224996
2007	230060
2008	214274
2009	229204

List 7: Tian jin

Year	Total Amount of Water Resources (100 million cu. m)
2002	3.67
2003	10.60
2004	14.31
2005	10.63
2006	10.11

2007	11.31
2008	18.30
2009	15.24

List 8: Tian jin

Year	Available water
2001	4.144
2002	2.7158
2003	7.844
2004	10.5894
2005	8.4652
2006	7.4814
2007	8.3694
2008	13.542
2009	11.2776