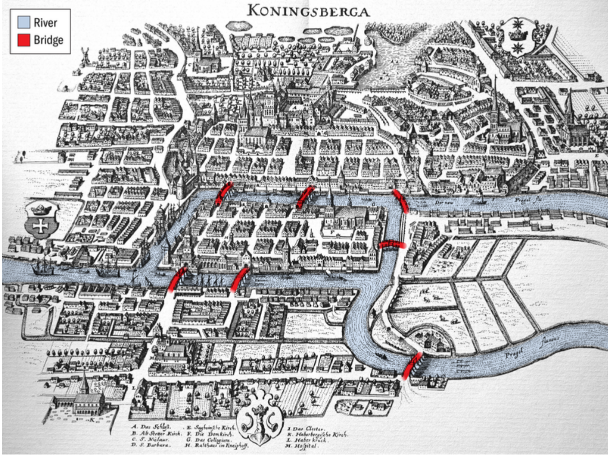
Königsberg Bridge Problem



1

Problem: devise a walk through the city that would cross each of those bridges once and only once.

[1] https://www.scientificamerican.com/article/how-the-seven-bridges-of-koenigsberg-spawned-new-math/

Leonhard Euler's Walk

Abstract the problem (Mathematical Modeling)

- Bridge size doesn't matter.
- Land shape is irrelevant.
- Connections are the key.

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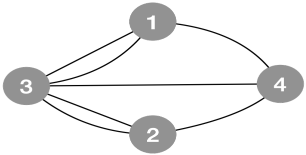

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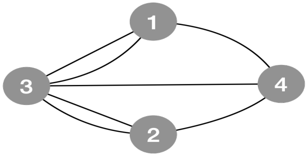
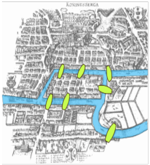
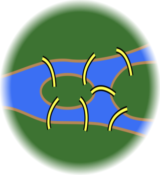


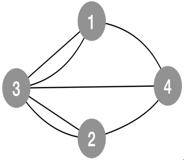
Negative resolution

- Whenever one enters a vertex by a bridge, one leaves the vertex by a bridge (Even degree).
- At most two endpoints.
- All four lands are touched by odd number of bridges (contradiction).

From life to math







[2]

[2] https://en.wikipedia.org/wiki/Seven_Bridges_of_Königsberg

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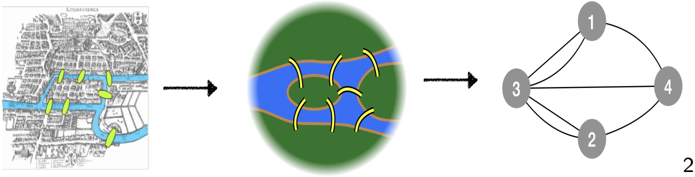

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Observation

- It can be any graph connected with arbitrary edges.
- Does Euler path exist? (can be a circle.)
- If so, what are the common conditions?

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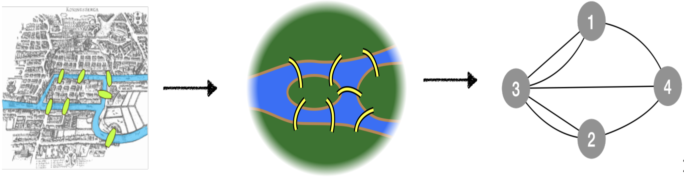

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Solution and beyond (Euler's Theorem)

- A connected graph has an Euler cycle if and only if every vertex has even degree.

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Eulerian Path (back to life!)

Real world applications

If there are nodes of odd degree, then any Eulerian path will start at one of them and end at the other. (the first theorem of graph theory.)

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If there are nodes of odd degree, then any Eulerian path will start at one of them and end at the other. (the first theorem of graph theory.)

- **Network routing**: finding an Eulerian path can help ensure efficient routing and minimize congestion.
- **Circuit design**: use Eulerian path to ensure efficient and error-free circuits.
- **DNA sequencing**: Eulerian paths can help assemble the short DNA fragments into longer contiguous sequences.
- **Robotics**: finding Eulerian path to avoid collisions with obstacles.

Q & A