Size of Sets

$$S_1 = \{3, 2, 6\} \quad S_2 = \{1, a\} \quad S_3 = \{2, a, d\}$$

Size of Sets

$$S_1 = \{3, 2, 6\} \quad S_2 = \{1, a\} \quad S_3 = \{2, a, d\}$$

What are the size of $S_1 \cap S_2$, $S_1 \cap S_3$, $S_2 \cap S_3$, $S_1 \cup S_2 \cup S_3$

Size of Sets

$$S_1 = \{3, 2, 6\} \quad S_2 = \{1, a\} \quad S_3 = \{2, a, d\}$$

What are the size of $S_1 \cap S_2$, $S_1 \cap S_3$, $S_2 \cap S_3$, $S_1 \cup S_2 \cup S_3$

- $|S_1 \cap S_2| = |\emptyset| = 0$
- $|S_1 \cap S_3| = |\{2\}| = 1$
- $|S_2 \cap S_3| = |\{a\}| = 1$
- $|S_1 \cup S_2 \cup S_3| = |\{3, 2, 6, 1, a, d\}| = 6$

Size of Product Set $S_1 \times S_2$

Size of Sets

$$S_1 = \{3, 2, 6\} \quad S_2 = \{1, a\} \quad S_3 = \{2, a, d\}$$

What are the size of $S_1 \cap S_2$, $S_1 \cap S_3$, $S_2 \cap S_3$, $S_1 \cup S_2 \cup S_3$

- $|S_1 \cap S_2| = |\emptyset| = 0$
- $|S_1 \cap S_3| = |\{2\}| = 1$
- $|S_2 \cap S_3| = |\{a\}| = 1$
- $|S_1 \cup S_2 \cup S_3| = |\{3, 2, 6, 1, a, d\}| = 6$

Size of Product Set $S_1 \times S_2$

- Two steps in total: Pick an element from S_1, then pick an element from S_2.

Size of Sets

$$S_1 = \{3, 2, 6\} \quad S_2 = \{1, a\} \quad S_3 = \{2, a, d\}$$

What are the size of $S_1 \cap S_2$, $S_1 \cap S_3$, $S_2 \cap S_3$, $S_1 \cup S_2 \cup S_3$

- $|S_1 \cap S_2| = |\emptyset| = 0$
- $|S_1 \cap S_3| = |\{2\}| = 1$
- $|S_2 \cap S_3| = |\{a\}| = 1$
- $|S_1 \cup S_2 \cup S_3| = |\{3, 2, 6, 1, a, d\}| = 6$

Size of Product Set $S_1 \times S_2$

- Two steps in total: Pick an element from S_1, then pick an element from S_2.
- First step: $|S_1|$ choices.

Size of Sets

$$S_1 = \{3, 2, 6\} \quad S_2 = \{1, a\} \quad S_3 = \{2, a, d\}$$

What are the size of $S_1 \cap S_2$, $S_1 \cap S_3$, $S_2 \cap S_3$, $S_1 \cup S_2 \cup S_3$

- $|S_1 \cap S_2| = |\emptyset| = 0$
- $|S_1 \cap S_3| = |\{2\}| = 1$
- $|S_2 \cap S_3| = |\{a\}| = 1$
- $|S_1 \cup S_2 \cup S_3| = |\{3, 2, 6, 1, a, d\}| = 6$

Size of Product Set $S_1 \times S_2$

- Two steps in total: Pick an element from S_1, then pick an element from S_2.
- First step: $|S_1|$ choices.
- Second step: $|S_2|$ choices.

Size of Sets

$$S_1 = \{3, 2, 6\} \quad S_2 = \{1, a\} \quad S_3 = \{2, a, d\}$$

What are the size of $S_1 \cap S_2$, $S_1 \cap S_3$, $S_2 \cap S_3$, $S_1 \cup S_2 \cup S_3$

- $|S_1 \cap S_2| = |\emptyset| = 0$
- $|S_1 \cap S_3| = |\{2\}| = 1$
- $|S_2 \cap S_3| = |\{a\}| = 1$
- $|S_1 \cup S_2 \cup S_3| = |\{3, 2, 6, 1, a, d\}| = 6$

Size of Product Set $S_1 \times S_2$

- Two steps in total: Pick an element from S_1, then pick an element from S_2.
- First step: $|S_1|$ choices.
- Second step: $|S_2|$ choices.
- In total: $|S_1 \times S_2| = |S_1| \times |S_2|$.

Size of Power Set

The size of power set of S, $|S| = k$

we have k elements to choose from.

Size of Power Set

The size of power set of S, $|S| = k$

we have k elements to choose from.

- Choices for the first element: 2.

Size of Power Set

The size of power set of S, $|S| = k$

we have k elements to choose from.

- Choices for the first element: 2.
- Choices for the second element: 2.

Size of Power Set

The size of power set of S, $|S| = k$

we have k elements to choose from.

- Choices for the first element: 2.
- Choices for the second element: 2.
- ...

Size of Power Set

The size of power set of S, $|S| = k$

we have k elements to choose from.

- Choices for the first element: 2.
- Choices for the second element: 2.
- ...
- Choices for the k^{th} element: 2.

Size of Power Set

The size of power set of S, $|S| = k$

we have k elements to choose from.

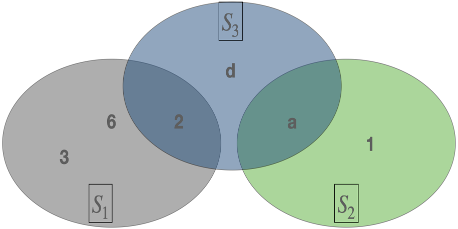
- Choices for the first element: 2.
- Choices for the second element: 2.
- ...
- Choices for the k^{th} element: 2.
- Final count: 2^k

Inclusion-Exclusion Principle

$|S_1 \cup S_2 \cup S_3| = |\{3, 2, 6, 1, a, d\}| = 6 \to$ General Cases?

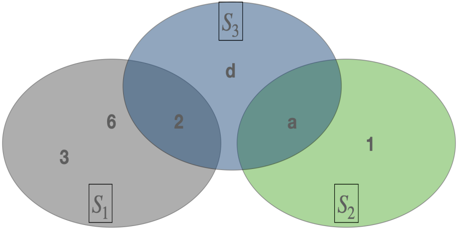
Inclusion-Exclusion Principle

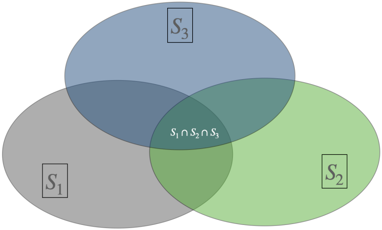
$|S_1 \cup S_2 \cup S_3| = |\{3, 2, 6, 1, a, d\}| = 6 \rightarrow$ General Cases?



Inclusion-Exclusion Principle

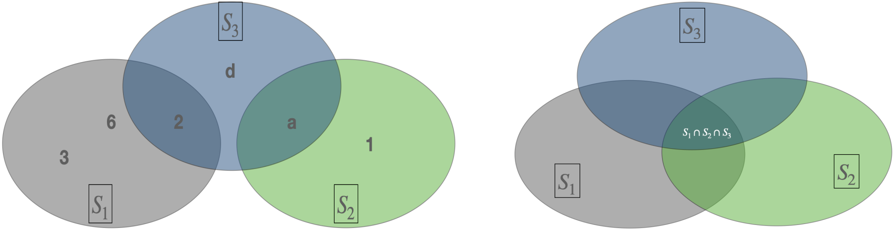
$|S_1 \cup S_2 \cup S_3| = |\{3, 2, 6, 1, a, d\}| = 6 \rightarrow$ General Cases?





Inclusion-Exclusion Principle

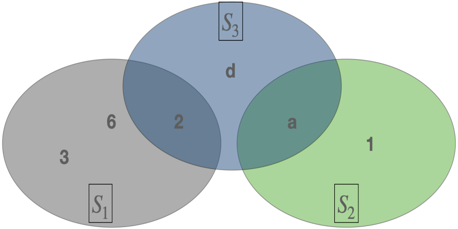
$|S_1 \cup S_2 \cup S_3| = |\{3, 2, 6, 1, a, d\}| = 6 \rightarrow$ General Cases?

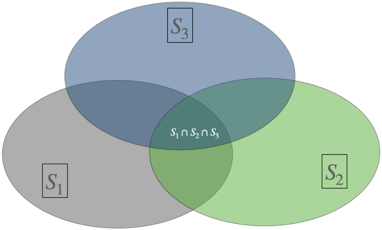


$$|S_1 \cup S_2 \cup S_3| = |S_1| + |S_2| + |S_3| - |S_1 \cap S_2| - |S_2 \cap S_3| - |S_1 \cap S_3| + |S_1 \cap S_2 \cap S_3|$$

Inclusion-Exclusion Principle

$|S_1 \cup S_2 \cup S_3| = |\{3, 2, 6, 1, a, d\}| = 6 \rightarrow$ General Cases?





$$|S_1 \cup S_2 \cup S_3| = |S_1| + |S_2| + |S_3| - |S_1 \cap S_2| - |S_2 \cap S_3| - |S_1 \cap S_3| + |S_1 \cap S_2 \cap S_3|$$

$$\begin{aligned} |S_1 \cup \ldots \cup S_k| &= |S_1| + \ldots + |S_k| \\ &- |S_1 \cap S_2| - \ldots - |S_{k-1} \cap S_k| \\ &+ |S_1 \cap S_2 \cap S_3| + \ldots + |S_{k-2} \cap S_{k-1} \cap S_k| \\ &\ldots \end{aligned}$$

Why counting sets matter?

Why counting sets matter?

- Algorithm/program analysis, complexity, how many steps in the loop?

Why counting sets matter?

- Algorithm/program analysis, complexity, how many steps in the loop?
- Optimization, how much computing resource required.

Why counting sets matter?

- Algorithm/program analysis, complexity, how many steps in the loop?
- Optimization, how much computing resource required.
- Database query and indexing.

Why counting sets matter?

- Algorithm/program analysis, complexity, how many steps in the loop?
- Optimization, how much computing resource required.
- Database query and indexing.
- Graph theory, number of vertices and edges.

Why counting sets matter?

- Algorithm/program analysis, complexity, how many steps in the loop?
- Optimization, how much computing resource required.
- Database query and indexing.
- Graph theory, number of vertices and edges.
- ...

Q & A