Proof states and deduction rules

$$p ::= \mathcal{A} \mid \top \mid \bot \mid \neg p \mid p_1 \wedge p_2 \mid p_1 \vee p_2 \mid p_1 \rightarrow p_2 \mid \forall x.p \mid \exists x.p$$

Proof state is a set of assumptions and goal proposition that are described in first order logic, to prove.

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Examples of $\Gamma \vdash p$

- Program state: $(x > 1) \wedge (y > 1) \vdash x + y > 0$
- Natural world:
  sky is pink; cat is a dancer... $\vdash$ The cat is dancing under pink sky.

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Propositions are expressed with our first order logic, there are some rules formally guide us to perform the proof, introduction rules in the goal proposition, elimination rules in the assumption.

Conjunction rules

intro-$\wedge$

$$\frac{\Gamma \vdash p_1 \ \wedge \ \Gamma \vdash p_2}{\Gamma \vdash p_1 \wedge p_2}$$

- Γ: Bob likes ice cream (p_1); Bob likes hotpot (p_2)...
- $p_1 \wedge p_2$: Bob likes ice cream and hotpot.

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elim-$\wedge$

$$\frac{p_1, \Gamma \vdash p(left) \ \vee \ p_2, \Gamma \vdash p(right)}{\Gamma(p_1 \wedge p_2) \vdash p}$$

- Γ: Alice is drinking coffee and listening to the music...
- p: Alice is listening to the music.

Implication rules

intro-$\rightarrow$

$$\frac{p_1, \Gamma \vdash p_2}{\Gamma \vdash p_1 \rightarrow p_2}$$

- Γ: I'm studying discrete structures at 8:30 am ...
- $p_1 \rightarrow p_2$: if it's 8:30 am(p_1), I'm studying discrete structures(p_2).
- p_1, Γ: It's 8:30 am; I'm studying discrete structures at 8:30 am...

Implication rules

intro-$\longrightarrow$

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elim-$\longrightarrow$

$$\frac{\Gamma \vdash p_1 \quad p_2, \Gamma \vdash p}{\Gamma(p_1 \rightarrow p_2) \vdash p}$$

- Γ: The floor is wet when it's raining ...
- p: The floor is wet.
- : How can you prove my cliam p is true or false?

Disjunction rules

intro-∨

$$\frac{\Gamma \vdash p_1(left) \;\; \vee \;\; \Gamma \vdash p_2(right)}{\Gamma \vdash p_1 \vee p_2}$$

- Γ: He can sleep anytime ...
- $p_1 \vee p_2$: He can sleep in the morning or in the evening.
- $\Gamma \vdash p_1(left)$: ?

Disjunction rules

intro-$\vee$

$$\frac{\Gamma \vdash p_1(left) \ \vee \ \Gamma \vdash p_2(right)}{\Gamma \vdash p_1 \vee p_2}$$

- Γ: He can sleep anytime ...
- $p_1 \vee p_2$: He can sleep in the morning or in the evening.
- $\Gamma \vdash p_1(left)$: ?

$\Gamma \vdash p_1(left)$: He can sleep anytime $\vdash$ He can sleep in the morning.

Disjunction rules

intro-$\vee$

$$\frac{\Gamma \vdash p_1(left) \;\vee\; \Gamma \vdash p_2(right)}{\Gamma \vdash p_1 \vee p_2}$$

- Γ: He can sleep anytime ...
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- $\Gamma \vdash p_1(left)$: ?

$\Gamma \vdash p_1(left)$: He can sleep anytime $\vdash$ He can sleep in the morning.

elim-$\vee$

$$\frac{p_1, \Gamma \vdash p \;\wedge\; p_2, \Gamma \vdash p}{\Gamma(p_1 \vee p_2) \vdash p}$$

Top, Bottom, Negation and Quantification rules

Check readings.

Q & A