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A mathematical framework for understanding uncertainty.

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 $\begin{array}{l} X:\Omega \to T \\ X:\Omega \to R \end{array}$

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Discrete Probability Distribution

Bernoulli distribution, binomial distribution, Poisson distribution, and geometric distribution.

Binomial distribution

• The number of successes in a fixed number of independent trials: $\Pr(k) = \binom{n}{k} p^k (1-p)^{n-k}$

Discrete Probability Distribution

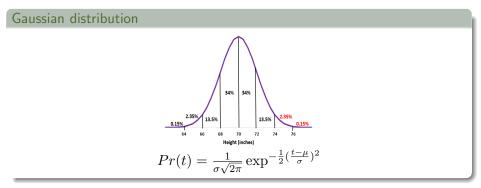
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Binomial distribution

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E(x) = np

Continuous Probability Distribution

Uiform distribution, normal (Gaussian) distribution, exponential distribution, and beta distribution.



Q & A